$\Theta_i^*(r, 0, \tau), \Theta_i^{**}(r, 0, \tau)$, T_0, excess temperatures on the surface of the semiinfinite body in the limiting cases for R_1 and R_2 (see text) and the initial temperature, respectively.

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CALCULATION OF AN OPTICAL SYSTEM WITH A HOLLOW MIRROR LIGHTGUIDE AND DIAPHRAGMS FOR PHOTOELECTRIC DEVICES

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A calculation method and nomograms are presented for optical systems with a hollow mirror cylindrical lightguide, input and output diaphragms, and a radiation receiver.

Hollow mirror lightguides [1, 2] are now being used in photoelectric equipment, especially pyrometers, together with lenses, mirrors, lightguides made of opticaly transparent materials, and other elements. The hollow mirror guides are nonselective, simple in construction, convenient in use, have high mechanical strength, and are low in cost. However, no methods are available for calculation of an optical system with hollow lightguides interacting with other elements — diaphragms, radiation receivers, lenses, etc.

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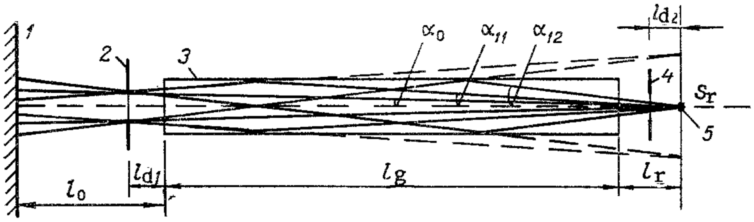

Fig. 1. Diagram of ray passage through lightguide with diaphragms: 1) object; 2) input diaphragm; 3) hollow cylindrical lightguide; 4) output diaphragm; 5) radiation receiver.

We will consider an optical system consisting of a hollow cylindrical mirror lightguide with round diaphragms at the input and output and a radiation receiver. An extended isothermal diffusely radiating body is located ahead of the lightguide.

Depending on the path which they traverse in the cylindrical lightguide, rays can conveniently be classified as meridional or oblique [1]. Meridional rays lie in planes passing through the axis of the lightguide and do not depart from such planes. For example, all rays from a source of any dimensions which pass through the lightguide and fall upon a point receiver located on the optical axis will be meridional.

Rays which do not intersect the lightguide axis are oblique. Oblique rays propagate along broken spiral lines, with their projections on the cross section of the cylindrical lightguide forming chords d, all of the same length. The smallest distance r from the guide axis to a chord remains constant. The number of reflections experienced by an oblique ray will be d_g/d times greater than for a meridional ray [1]. Considering that $d = \sqrt{d^2g^2 - 4r^2}$, we find that $d_g/d = [1 - 2r/d_g]^{-1/2}$. Since the largest value of 2r is equal to the diameter of the receiver sensitive area $2r = d_r$, it follows that if $(d_r/d_g)^2 \ll 1$, the number of reflections for oblique and meridional rays, and thus their energy losses within the guide, will be the same, so that they may be considered quasimeridional. We will limit our examination to the case where receiver dimensions satisfy this condition.

It follows from Fig. 1 that rays from the object traversing the lightguide without reflection (zeroth zone) are contained within a cone with meridional angle

$$\alpha_0 = \operatorname{arc\,tg} \frac{d_{d1}}{2(l_{d1} + l_g + l_r)}. \tag{1}$$

Rays falling on the radiation receiver after a single reflection (first zone) propagate within the solid angle included between two cones with apex angles α_{11} and α_{12}, where

$$\alpha_{11} = \operatorname{arc\,tg} \frac{d_g - d_{d1}/2}{l_{d1} + l_g + l_r}, \qquad \alpha_{12} = \operatorname{arc\,tg} \frac{d_g + d_{d1}/2}{l_{d1} + l_g + l_r}. \tag{2}$$

After twofold reflection (second zone):

$$\alpha_{21} = \operatorname{arc\,tg} \frac{2d_g - d_{d1}/2}{l_{d1} + l_g + l_r}, \qquad \alpha_{22} = \operatorname{arc\,tg} \frac{2d_g + d_{d1}/2}{l_{d1} + l_g + l_r}. \tag{3}$$

Rays of the n-th zone arrive at the receiver from between two cones with apex angles α_{n1} and α_{n2}:

$$\alpha_{n1} = \operatorname{arc\,tg} \frac{nd_g - d_{d1}/2}{l_{d1} + l_g + l_r}, \qquad \alpha_{n2} = \operatorname{arc\,tg} \frac{nd_g + d_{d1}/2}{l_{d1} + l_g + l_r}. \tag{4}$$

Zeroth-zone rays (direct rays) depart from a disk on the object. Rays of the first and subsequent zones (Fig. 1) can be regarded as departing from imaginary sources in the form of corresponding concentric rings in the object plane. We will find the value of the flux incident on the receiver from the disk (zeroth zone) and each of these rings.

If the object has a diffusely radiating surface, then the energetic brightness of its radiation is identical in all directions and equal by definition to [3]:

$$I_0 = \frac{d\Phi}{d\omega\, dS_0 \cos\beta}.$$

Whence

$$d\Phi = I_0 d\omega\, dS_0 \cos\beta = I_0 \frac{S_r \cos\alpha}{r^2} \cos\beta = \pi I_0 S_r \frac{\cos\alpha \cos\beta}{\pi r^2} = \pi I_0 S_r d\varphi_{r0}.$$

The flux incident on the receiver S_r from a disk of area S_0 will equal:

$$\Phi = \int\limits_{S_0} d\Phi = \pi I_0 S_r \int\limits_{S_0} d\varphi_{r0} = \pi I_0 S_r \varphi_{r0}.$$

Since the distance between receiver and disk is $\ell_0 + \ell_g + \ell_r$, the local angular coefficient of radiation of an elementary receiver on the disk can be defined by the expression [4]: $\varphi_{r0} = R_0^2/[R_0^2 + (\ell_0 + \ell_g + \ell_r)^2] = \sin^2\alpha_0$.

The local angular coefficient on the ring of the n-th zone, the edges of which are visible from the receiver at angles α_{n2} or α_{n1} will be:

$$\varphi_{rn} = \sin^2\alpha_{n2} - \sin^2\alpha_{n1}.$$

The flux incident on the receiver from the zeroth zone (direct rays) is:

$$\Phi_0 = \pi I_0 S_r \varphi_{r0} = \pi I_0 S_r \sin^2\alpha_0.$$

The flux incident from the n-th imaginary ring (n-fold reflected rays, n-th zone) is:

$$\Phi_n = \rho^n \pi I_0 S_r \varphi_{rn} = \rho^n \pi I_0 S_r (\sin^2\alpha_{n2} - \sin^2\alpha_{n1}). \tag{5}$$

The total flux from the object passing through the lightguide and reaching the receiver is

$$\Phi = \Phi_0 + \sum_{n=1}^{m} \Phi_n = \pi I_0 S_r [\sin^2\alpha_0 + \sum_{n=1}^{m} \rho^n (\sin^2\alpha_{n2} - \sin^2\alpha_{n1})].$$

As follows from purely geometric considerations, if the role of the aperture diaphragm is played by the output orifice of the lightguide, then $m = m_0$, where

$$m_0 = l_g/2l_r; \tag{6}$$

if the aperture diaphragm is an input diaphragm, then $m = m_1$, where

$$m_1 = l_g(1 + b_1)/2l_{d1}, \tag{7}$$

where

$$b_1 = d_{d1}/d_g;$$

while if the aperture is limited by an output diaphragm, then $m = m_2$, where

$$m_2 = [b_2(l_g + l_r) - l_{d2}]/2l_{d2}, \quad b_2 = d_{d2}/d_g. \tag{8}$$

In practice, it is necessary to calculate m_0, m_1, m_2, and take the smallest value for m.

Using Eqs. (1), (4), we find the values of the sines of the angles α_0, α_{n1}, α_{n2}, substitute in Eq. (5), introduce the notation $a = (\ell_{d1} - \ell_g + \ell_r)/d_g$, and, considering that $a \gg 1$, after some manipulations we obtain

$$\Phi = \frac{\pi I_0 S_r b_1^2}{4a^2}\left[1 + \sum_{n=1}^{m} \frac{8n\rho^n}{b_1\left[1 + \left(\frac{n}{a}\right)^2\right]^2}\right]. \tag{9}$$

In the absence of a lightguide, where the flux is limited only by the input diaphragm, only rays of the zeroth zone arrive at the receiver

$$\Phi_0 = \frac{\pi I_0 S_r b_1^2}{4a^2}.$$

The presence of the lightguide increases the flux by a factor of y_1 times:

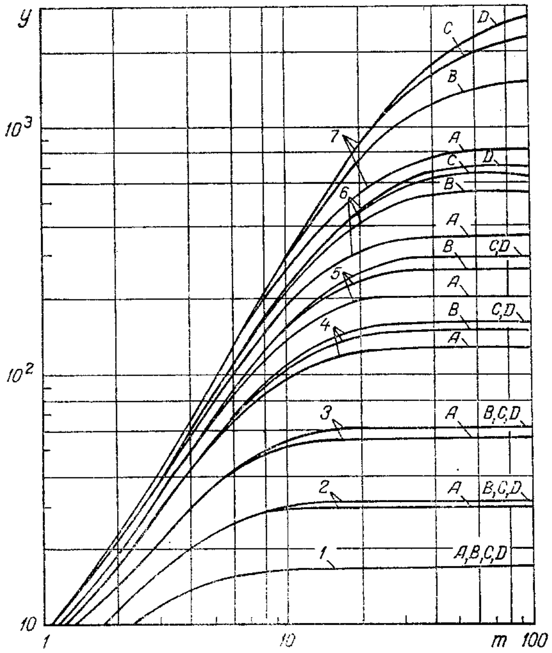

Fig. 2. Function $y = \Phi/\Phi_0$ vs number of reflections m in lightguide for various values of reflection coefficient ρ and parameter a: 1-7) ρ = 0.5, 0.6, 0.7, 0.8, 0.85, 0.9, 0.95; parameter a = 25, 50, 100, 200 for curves A, B, C, D.

$$y_1 = \Phi/\Phi_0 = 1 + \sum_{n=1}^{m} \frac{8n\rho^n}{b_1\,[1 + (n/a)^2]^2}. \tag{10}$$

In the absence of an input diaphragm ($b_1 = 1$, $\ell_{d1} = 0$)

$$y = 1 + \sum_{n=1}^{m} \frac{8n\rho^n}{[1 + (n/a)^2]^2}. \tag{11}$$

The dimensionless function y can be called the lightguide efficiency or its "amplification" coefficient. It follows from curves calculated with Eq. (11) and depicted in Fig. 2 that the lightguide efficiency y depends significantly on the reflection coefficient of its inner surface, while the effect of the parameter a is significant only at large ρ.

Saturation appears in each curve after a certain number of reflections m = m', i.e., due to attenuation in lightguide the angular zones with m > m' produce practically no contribution to the flux incident on the receiver.

Each angular zone corresponds to a ring on the object from which radiation falls on the receiver. As follows from Fig. 2 and Eq. (4), the maximum diameter of the n-th ring is equal to

$$D_n = d_{d1} + 2l_0\,\mathrm{tg}\,\alpha_{n2} = d_{d1} + \frac{2n + b_1}{a}\,l_0. \tag{12}$$

The relationships obtained may be used to calculate the field of view and radiation flux incident on the receiver in the presence of diaphragms or in the absence of one or both diaphragms. With no input diaphragm $\ell_{d1} = 0$, $d_{d1} = d_g$, $b_1 = 1$. The output diaphragm can only limit the possible number of reflections in the lightguide m_2 (Eq. (8)) and, thus, the flux and field of view. To speed calculations the graphs of Fig. 2 may be used as nomograms.

A device, for example, a pyrometer with hollow lightguide with or without diaphragms, can be calculated in the following sequence. First, we determine m, the maximum possible number of reflections of rays reaching the radiation receiver. To do this Eqs. (6)-(8) are used to calculate m_0, m_1, and m_2, and the smallest of these values is taken as m. We determine the parameter a for the given system, and knowing the reflection coefficient ρ, we use the corresponding curve of Fig. 2 to determine m', i.e., the point after which the curve becomes horizontal (saturated). We compare m and m', after which two variants are possible.

1. If m < m', then the angle of view of the device is determined by the angular zone with number m. The diameter of the viewed spot (field of view) will not exceed

$$D_m = d_{d1} + \frac{2m + b_1}{a} l_0, \quad (13)$$

where m is the smallest of the values m_0, m_1, and m_2, determined by Eqs. (6)-(8).

2. If $m \geq m'$, then, as follows from Eq. (2), beginning with m', the contribution of subsequent angular zones to the flux is practically zero (the function y becomes horizontal), so that the angle of view of the device is determined not by the angular zone with number m, but the zone with number m', which can be determined from Fig. 2.

The diameter of the spot viewed can be determined with Eq. (13), where in place of m we substitute the value m'.

The radiation flux reaching the receiver is determined with Eq. (9), or by using the nomograms of Fig. 2, we determine y, and then

$$\Phi = \Phi_0 y_1 = \frac{\pi I_0 S_r b_1^2}{4a^2}\left[1 + \frac{y-1}{b_1}\right]. \quad (14)$$

In the absence of an input diaphragm $\ell_{d1} = 0$, $b_1 = 1$, and then

$$\Phi \doteq \frac{\pi I_0 S_r}{4a^2} y.$$

An output diaphragm produces the same effect as shortening the lightguide on the receiver side by an amount $\Delta\ell_g = \ell_{d2}/b_2 - \ell_r$, allowing reduction in the energy flux and field of view. The calculation with or without the output diaphragm is the same, the role of the diaphragm in the latter case being played by the output orifice of the lightguide, so that $\ell_{d2} = \ell_r$, $d_{d2} = d_g$, $b_2 = 1$, $m_2 = m_0$.

Experimental studies were performed with an apparatus consisting of an oxidized metal disk with heater, diaphragm, nickel hollow cylindrical lightguide, radiant flux modulator, FD-3A germanium photodiode, and measurement circuitry. The field of view of the system was determined for various geometric parameters. Comparison of experimental and calculated data revealed a divergence of less than 5%, which was within the limits of experimental uncertainty. In particular, $D_m = 28 \pm 1$ mm for $d_r = 1$ mm, $d_g = 8$ mm, $\ell_g = 400$ mm, $\ell_r = 20$ mm, $d_{d1} = 8$ mm, $\ell_{d1} = 10$ mm, $\ell_0 = 50$ mm.

Thus, calculation of a pyrometer or other device with hollow mirror lightguide with or without input and output diaphragms can be performed using the curves presented in Fig. 2. Those curves and Eqs. (6)-(8) define the number of the largest angular zone which still produces a contribution to the flux reaching the receiver, and this number and Eq. (13) define the size of the spot viewed. The value of the function y and Eq. (14) then determine the value of the flux incident on the radiation detector.

NOTATION

ℓ_g, d_g, length and inner diameter of lightguide; d_{d1}, d_{d2}, diameter of first (input) and second (output) diaphragms; ℓ_{d1}, ℓ_{d2}, distances from corresponding diaphragms to nearest face of lightguide and radiation receiver; ℓ_r, distance from radiation receiver to output end of lightguide; ℓ_0, distance from object to input end of lightguide; ρ, reflectivity of inner surface of lightguide; I_0, energetic brightness of object radiation; Φ, radiant flux passing from object through lightguide to receiver; S_0, object area; $d\phi_{r0}$, angular radiation coefficient of elementary receiver with respect to element of object; ϕ_{r0}, local angular radiation coefficient of elementary receiver with respect to the disk; ϕ_{rn}, local angular radiation coefficient of receiver with respect to n-th ring zone; m, maximum possible number of reflections for rays reaching the radiation receiver; D_m, diameter of visible spot; α, angle between direction toward object element dS_0 and normal to plane of receiver S_r; r, distance between planes S_0 and S_r; β, angle between direction to receiver and normal to plane dS_0; $d\omega$, solid angle over which receiver area is visible from element dS_0.

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